

Technical University of Lodz

Department of Semiconductor and Optoelectronics Devices

[WWW.DSOD.PL](http://WWW.DSOD.PL)

## **ELECTRONIC MEASUREMENT LAB.**

Experiment No 1

ALGORITHMS of DIGITAL SIGNAL PROCESSING  
for MEASUREMENT PURPOSES

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**Goal:**

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The goal of this experiment is to familiarise students with basic discrete signal transformations used in measurement systems with Analogue-to-Digital Converters. Theoretical and practical aspects of digital and analogue filters as well as signal spectrum are briefly presented.

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**SPECIFICATION OF USED INSTRUMENTS:**

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The following instruments and software are used:

**Instruments**

1. Signal generator DDS type DF1410
2. 2-channel Digital Oscilloscope type RIGOL 1052E (FFT module included)
3. 12-bit Multifunction USB Module USB-4711A (Advantech)
4. Digital Sampling Multimeter RIGOL DM3051
5. Student's „DSP-Kit“

**Software:**

1. Software supporting Multifunction USB Module USB-4711A
2. Data Processing supporting student's DSP-Kit
3. Microsoft EXCEL with data analysis facilities

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## THEORY

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Algorithms of Digital Signal Processing incorporated in as elements of the measuring devices are one of the most important blocks of measuring chains in today's instruments. Contemporary measuring instruments characterise minimal number of analogue elements preceding analogue to digital converter block still we observe the tendency to reduce analogue elements. DSP algorithms are a greater help to such approach. As a result of digitalisation of analogue signals we receive set of digital sets upon which we proceed mathematical operations. All operations are now in a discrete form. One of such replacement of analogue operation is an integration, which is replaced by summation operation.

Algorithms of digital signal filtering and Discrete Fourier Transform) play a special key role and are widely applied.

### Discrete Fourier Transform

DFT is mainly used to spectrum analysis of signals in measuring instruments but application of DFT is far broader than only for spectrum analysis.

DFT make possible:

- Simple signal representation by representing signals in frequency domain instead of in time domain (like polyharmonic signals)
- analysis of results of signal processing in analogue blocks which were performed before spectrum analysis (filtration interpolation)
- analysis of signal response of different objects
- and finally conclusions from spectrum analysis can be used for decision making in adaptive algorithms, voice analysis and recognition, identification processes and signal transmission

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text from chapter of:

**Joseph McGhee, Wlodek Kulesza, M. Jerzy Korczyński, I. A. Henederson, Measurement Data Handling Theoretical Technique, Published by Technical University of Lodz, printed by: ACGM LODART S. A. Łódź, 2001, ISBN 83-7283-007-X, pages 267 vol. 1**

Frequency analysis is the process of calculating numerical values for signal parameters in the frequency domain. It enables both the information content embedded in the signal and any contaminating interference to be represented in a concise form. The aim is to distinguish the information bearing part of the signal from the noise and to separate them. *Fourier Analysis* is the most important tool for this purpose. It provides the key which relates signals in the time and frequency domains.

Data collecting can be done only in a certain time period. That data collection period is called the measurement window or just window. If we collect data of periodical signal for example electrical voltage or current, but not of such signals are periodical, and if we choose the length of measurement window  $T_W$  equal to  $T$  period of sampled signal, and the sampling period is equal to:

$T_S = T_W / M$  we get  $M$  samples, which represent the signal and the samples are of values:  $x(0), x(T_S), x(2T_S), \dots, x[(M-1)T_S]$ . Based on values of the collected samples, it is possible to complete  $M$  equations, which comprehend the Fourier series components (Equ 3.1). Solving such set of equations, it is possible to get:  $a_0=A_0$  and  $N = M/2 - 1$  harmonic components, described by  $a_n, b_n$  coefficients or by amplitude  $A_n$  and phase  $\varphi_n$

$$x(t) = a_0 + \sum_{n=1}^N (a_n \cos n\omega t + b_n \sin n\omega t) = A_0 + \sum_{n=1}^N A_n \sin(n\omega t + \varphi_n) \quad (3.1)$$

In practice, the coefficients of Fourier series, are obtained by transformation procedure, of  $M$  point collection of samples  $x(mT_S)$  into  $M$  point discrete series in frequency domain (Discrete Fourier Transform DFT)

$$\underline{X}_n = \underline{X}(nf_w) = \sum_{m=0}^{M-1} x(mT_S) \cdot e^{-j\frac{2\pi mn}{M}} = \underline{X}(0), \underline{X}(f_w), \dots, \underline{X}[(M-1)f_w] \quad (3.2)$$

where:  $n = 0, 1, 2, \dots, M-1, f_w = 1/T_W$ .

The Inverse Fourier Transform allows to return from frequency domain to time domain, and it is given by formula (3.3)

$$x(mT_S) = \frac{1}{M} \sum_{n=0}^{M-1} \underline{X}(nf_w) \cdot e^{j\frac{2\pi mn}{M}} \quad (3.3)$$

To obtain the amplitudes and phases of  $N$  harmonics, it is enough to know of the half of elements of series of elements in frequency domain  $\underline{X}(nf_w)$ , as the second half of elements represent complex conjugate values to values of the first half of obtained values with an exception of element  $\underline{X}(M/2)$ , which is equal to zero ( $\underline{X}(M/2)=0$ ):

$$a_0 = A_0 = \frac{1}{M} \underline{X}(0) \quad (3.4)$$

$$a_n = \frac{1}{M} (\underline{X}_n + \underline{X}_{M-n}) = \frac{2}{M} \text{Re}(\underline{X}_n) \quad (3.5)$$

$$b_n = \frac{j}{M} (\underline{X}_n - \underline{X}_{M-n}) = -\frac{2}{M} \text{Im}(\underline{X}_n) \quad (3.6)$$

$$A_n = \frac{2}{M} |\underline{X}_n| \quad (3.7)$$

$$\varphi_n = -\arg(\underline{X}_n) \quad (3.8)$$

where:  $n = 1, 2, \dots, N$ .

If the number of samples is a power of 2,  $M = 2^c$ , it is possible to apply a Fast Fourier Transform algorithm FFT, which allows to reduce time of computation significantly, so for example for  $M = 1024$  the reduction of number of multiplications in the algorithm is reduced by 100 times, what is resulting in significant reduction of time calculation.

Spectrum  $\underline{X}(nf_w)$  of the signal is presented in the form of two graphs: amplitude spectrum  $A_n(f)$  and phase spectrum  $\varphi_n(f)$ . Resolution of the spectra is denoted as  $f_w$  and for  $T_W = T = T_I$  is equal to basic harmonic  $f_w = 1/T_I = f_I$ . If the measurement

window comprise integer  $p$  periods of the signal  $T_w = pT_1$ , then  $f_w = 1/pT_1 = f_1/p$  decreases its value, and consequently the density of spectral lines increases. For such situation it is possible to measure sub and interharmonics. The width of measuring window  $T_w = MT_s = M/f_s$  increases, when the number of samples  $M$  increases, and/or  $f_s$  decreases, both of these parameters must be calculated in such a way, that in the measuring window, the integral  $p$  number of periods should be used. It means that the relation  $T_w = p/f_1 = M/f_s$  should be comply, and then the following equation determines sampling frequency  $f_s$ .

$$f_s = \frac{Mf_1}{p} = Mf_w \quad (3.9)$$

If the measuring window do not contain the integer number of periods, then it is not possible to measure precisely of harmonics and as the resolution of spectrum is not a sub multiplicity of basic harmonics and each harmonic is represented by several spectral lines (Fig. 3.1).

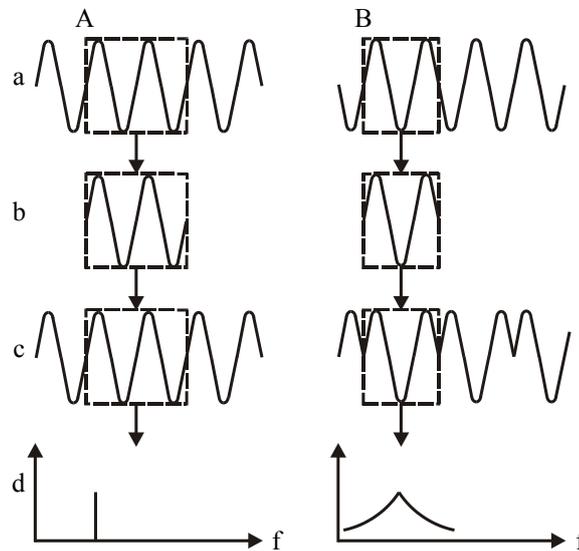


Fig.3.1 Influence of width of measuring window for spectrum shape: A – width of the window equal to two periods of the signal, B – width of the window not equal to two periods of the signal, a – signal under examination, b – part of signal in measuring window, c – signal used for FFT calculations, d – spectrum

### 3.2.3. Sampling theorem

According to Shannon-Kotelnikov theorem sampling of the signal do not cause loose of information if:

- The sampling frequency  $f_s$  is more than two times higher than the frequency of the highest harmonic in tested signal  $f_g$ ,
- The signal under test does not include harmonics of which frequency is higher than frequency equal to  $0.5 f_s$  (half of sampling frequency)

Usually:  $f_s = 2\lambda f_g$ , where  $\lambda > 1$  and is called over sampling coefficient.

If the second condition is not complied than any harmonics of frequency  $f$  higher than  $f_s/2$  would be transformed to the frequency  $f_p$  in the range:  $<0, f_s/2$ ) and may influence the value of harmonics carrying information in the range  $<0, f_s/2$ )

$$f_p = |f - if_s| \quad (3.10)$$

where:  $i$  is an integer number which comply the condition:

$$-f_s/2 < f - if_s < f_s/2 \quad (3.11)$$

Measured spectral component of frequency  $f_p$  may contain also harmonics of the following frequencies:  $f_s - f_p$ ;  $f_s + f_p$ ;  $2f_s - f_p$ ;  $2f_s + f_p$ ; ..., described by the following relation:

$$f = if_s \mp f_p \quad i = 1, 2, \dots \quad (3.12)$$

The effect of overlapping (reflections) in spectrum called *aliasing* is presented in graphical form in Fig. 3.2 in time and in Fig 3.3 in frequency domain. In Fig 3.2 are presented two signals: one of 150 Hz and the second of 250 Hz and if the sampling frequency is 200 Hz, then the signal of frequency 50 Hz is identified. The second drawing presents how harmonics reflect from the barrier of  $f_s/2$  and  $f=0$  if their frequencies are higher than  $f_s/2$  and are placed in the range  $0 - f_s/2$ .

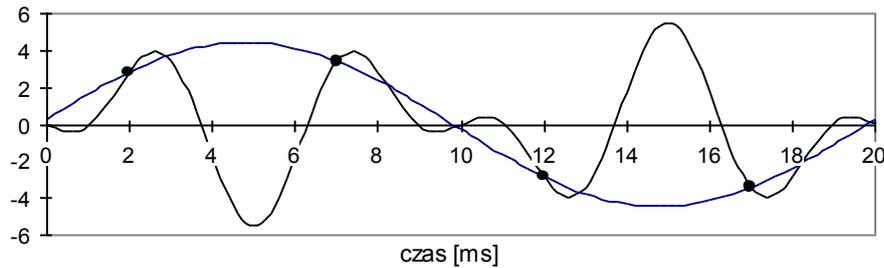


Fig 3.2. The sinusoidal signals of frequency 150 Hz and 250 Hz are sampled with the  $f_s=200$  Hz; then the sampled points show the frequency of 50 Hz ( $|150 - 200| = 50$  and  $|250 - 200| = 50$ )

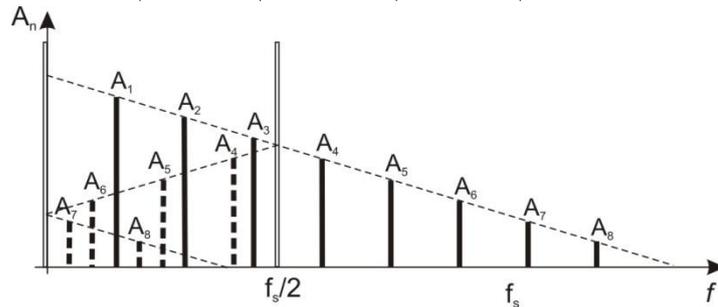


Fig. 3.3. Presentation of the phenomena of harmonic reflections

To avoid overlapping in spectrum, it is necessary to delete from analogue signal all frequencies over  $f > f_s/2$ . The analogue low band filters also called *anti-aliasing filters* are used. The other method is to use over sampling method with  $\lambda \gg 1$  (*over-sampling*). Then barrier of frequencies  $f_s/2$  is shifted to upper region. Sampled in such a way signal may be passed through digital low band filter and after significant reduction of samples, might be processed by FFT. Thanks to such operation the calculation procedure is faster and the spectrum resolution can be better (higher density of spectral lines).

Example:

In ADC used in acoustic signal analysis sampling frequency equals:  $f_s = 44,1$  kHz, what for highest audible frequency of  $f_g = 20$  kHz results that coefficient  $\lambda = 1,10$ . If microphone transducer converts audio signal up to 30 kHz, then harmonics from the range  $f_s/2 = 22,05$  kHz to 30 kHz are transferred to the range from  $|30 - 44,1| = 14,1$  kHz to  $|22,05 - 44,1| = 22,05$  kHz. These harmonics appears in audible range and disturb correct signals (voice). To reduce that effect it is worth to apply anti-aliasing filter of cutoff frequency 22 kHz.

To the opposite to over-sampling is under-sampling. Under-sampling is used in signal analysis for signals of small band  $\Delta f$  (carrying information) in comparison to carrier frequency  $f_0$ , for example: for signals which are amplitude modulated signals. To get an adequate resolution in  $\Delta f$  band it is necessary to lower sampling frequency to values, which are several times lower than  $f_0$ .  
end of texte from chapter 3.

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DFT from physical point of view is a transformation from time domain to frequency domain. In mathematical sense it is transformation from real to complex numbers.

DFT is resulting in conversion of one set of elements in time domain to frequency domain, and elements in time and spectrum line in frequency domain.

Inverse Discrete Fourier Transform also exists, so it is possible invers transformation: from frequency domain to time domain

A pair of transforms: DFT, IDFT are given by: (1 – 4):

Exponential form: of DFT in polar coordinates (1),

$$X(k) = \sum_{n=0}^{N-1} x(n) \cdot e^{-j2\pi nk / N} \quad (1)$$

Trigonometric form of DFT is given by (2),

$$X(k) = \sum_{n=0}^{N-1} x(n) \cdot [\cos(2\pi nk / N) - j \sin(2\pi nk / N)] \quad (2)$$

IDFT (exponential form) in polar coordinates is given by (3),

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) \cdot e^{j2\pi kn / N} \quad (3)$$

Trigonometric form of DFT is given by (4)

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) \cdot [\cos(2\pi kn / N) - j \sin(2\pi kn / N)] \quad (4)$$

where: :

n – number of sample and x(n) is a sample in time domain,

k – number of spectrum line X(k) spectrum Line in frequency domain,

N – number of iterations (number of calculations of DFT/IDFT)

Resolution  $f_k$  between spectral lines after DFT is calculated from (5) if k=1

$$f_k = k \left( \frac{f_s}{N} \right) \quad (5)$$

$f_k$  – frequency of k spectral line

Calibration of amplitude of spectrum for  $f = 0\text{Hz}$  (constant component in time domain) can be calculated according to 6,

$$X'_{am}(k) = \frac{X_{am}(k)}{N} \quad (6)$$

for other frequencies  $f \neq 0\text{Hz}$  (periodical component in time domain) according to formula given by 7.

$$X'_{am}(k) = \frac{X_{am}(k)}{N/2}$$

Properties of DFT:

LINEARTITY

$$x_1(n) \xleftrightarrow{DFT} X_1(k)$$

$$x_2(n) \xleftrightarrow{DFT} X_2(k)$$

$$a \cdot x_1(n) + b \cdot x_2(n) \xleftrightarrow{DFT} a \cdot X_1(k) + b \cdot X_2(k)$$

SYMETRY

$$x(n) \xleftrightarrow{DFT} X(k)$$

$$x(-n) \xleftrightarrow{DFT} X^*(k)$$

MULTIPLICATION

$$x_1(n) \cdot x_2(n) \xleftrightarrow{DFT} X_1(k) \otimes X_2(k)$$

CONVOLUTION

$$X_1(k) \otimes X_2(k) \xleftrightarrow{DFT} x_1(n) \cdot x_2(n)$$

SCALING

$$x(an) \xleftrightarrow{DFT} \frac{1}{|a|} X\left[\frac{k}{a}\right] \text{ dla } a > 0$$

DELAY

$$x(n-m) \xleftrightarrow{DFT} e^{-j2\pi nm} X(k)$$

MODULATION

IF modulation function is in the form:  $e^{j2\pi nm}$  that:

$$e^{j2\pi nm} \cdot x(n) \xleftrightarrow{DFT} X[k-m]$$

ORTOGONALITY

$$x(n) \xleftrightarrow{DFT} X(k) \quad \text{oraz} \quad X(k) \xleftrightarrow{IDFT} x(n)$$

CORELATION

$$z(n) = x_1(n) \otimes x_2(n-m) \xleftrightarrow{DFT} Z(k) = X_1(k) \cdot X_2^*(k)$$

PARSEVAL THEOREM (POWER)

$$\sum_n |x(n)|^2 \xleftrightarrow{DFT} \sum_k |X(k)|^2$$

The basic application of DFT in spectral analysis for metrological applications is to identify spectrum and frequency range in which power of energy is contained (is transmitted)

Correct identification of spectrum requires the following conditions:

- frequency of analogue signal must be of limited frequency range (frequency band) by applying anti-aliasing filter (low pas band filter) correlated to sampling frequency of AD converter ,
- analogue signal for DFT must be periodical, and signal for DFT must contain one or more periods.
- the frequency resolution of the signal spectrum in frequency domain should be chosen in such a way that spectrum should represent analogue signal in the best possible way (most possible information about analogue signal should be contained in spectrum of that signal) .

Fig. 1 to 4 are presenting sinusoidal signal and their complex form of spectrum (imaginary part of the spectrum composite), amplitude and phase (real part of the spectrum in this case equal to zero for each spectrum line).

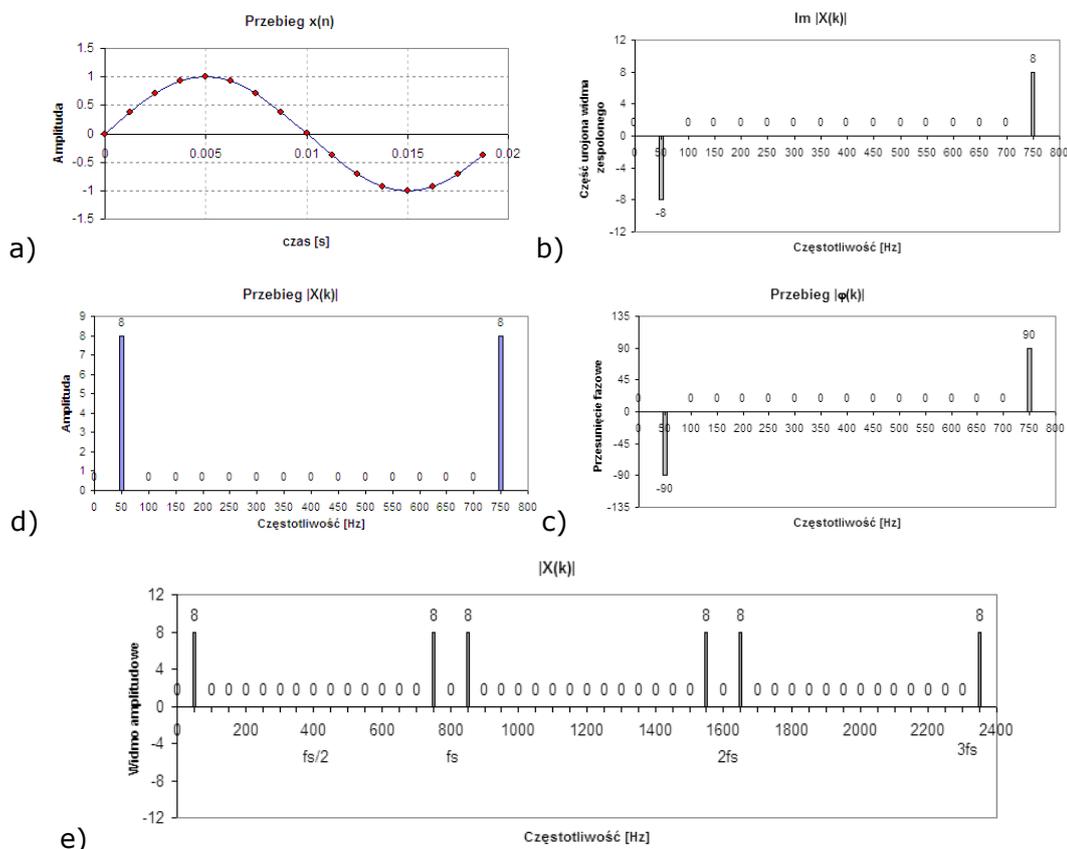


Fig.1-5. a) analogue sinusoidal signal of amplitude  $A=1$  and frequency  $f=50$  Hz. and samples in-time domain (red dots) – 16 samples, sampling frequency  $f_s=50 \cdot 16$  Hz = 800 Hz. b) imaginary part of complex number of spectrum part spectrum of complex

number within range of 0÷800Hz; c) modulus of complex numbers representing spectrum (amplitude spectrum) in the range 0÷800Hz; d) argument of complex number representing spectrum (phase) expressed in deg. frequency range 0÷800Hz; e) modulus, absolute value of complex number of spectrum presented in complex numbers (Spectrum of amplitude) in the frequency range 0÷2400Hz ( $0 \div 3 f_s$ ) presenting periodicity of DFT

### Algorithms of decimation and interpolation of signals in a discrete-time signal.

**Decimation** is a technique for reducing the number of samples in a discrete-time signal.

Sometimes it is necessary to adapt – to reduce a number of samples. by changing sampling frequency (lowering sampling frequency). The purpose is to fulfil some hardware requirements or to shorten time of calculations.

For that purpose is possible to use decimation operation it means to reduce number of samples in set of data by elimination of periodical samples from collected data from AD Converter (ADC converts analogue signal in a discrete-time signal form. A number of reduced of elements from set of digits  $x(n)$  is described by decimation factor,  $D$ . If decimation factor  $D=2$ , then every second sample is eliminated from set of  $x(n)$ . If decimation factor  $D=3$  then every third sample remains in the set. New sampling frequency is equal:  $f_{sn}=f_s/D$ . So the range of spectrum becomes lower and resolution of spectrum  $f_k$ , becomes lower as both are correlated to sampling frequency. In order to avoid aliasing effect after decimation, the decimation operation should be preceded by low pass filtering.

**Interpolation operation** is used to rise number of samples in software way, so the number of samples in a set  $x(n)$  increases. Increase of number of samples is without knowing of analogue function and is proceed using information from already sampled values register is set of  $x(n)$  earlier samples analogue signal with sampling frequency of  $f_s$ . The simplest method of interpolation is a linear interpolation by factor  $U = 2$ . In such case between two consecutive samples of set of  $x(n)$  a new sample is added. That new sample is calculated as mean average of two adjacent samples of  $x(n)$ . In general new sampling frequency is given by formula  $f_{sn}=U \cdot f_s$  and is just  $U$ -times higher than primary sampling frequency. The most popular method of interpolation is of discrete in time values of analogue signal is an interpolation by means of function described by sinc( $n$ ) function,  $\text{sinc}(n)=\sin(n)/(n)$ . Interpolation algorithm by sinc( $n$ ) function consists of two steps. The first step is a he interpolated set is fill out by zeros according to interpolating factor,  $U$ . If  $U=4$ , then, every fourth sample is from original set of samples, and three others are zeros:  $x'(n)=\{x(0),0,0,0,x(1),0,0,0,x(2),\dots\}$ . The second step is a convolution operation (consult: <http://en.wikipedia.org/wiki/Convolution>) with interpolated set of samples  $u(m)=\text{sinc}(m)$ . as the result of convolution, non-zeros samples

from interpolation are the sum of linear combination of original samples multiply by sequences of samples of interpolation samples sinc function..

In practice interpolating sequence is several times longer than  $U$  component. The interpolating sequence of sinc function of length  $m=19$  for  $U=4$  is given in Fig. 6

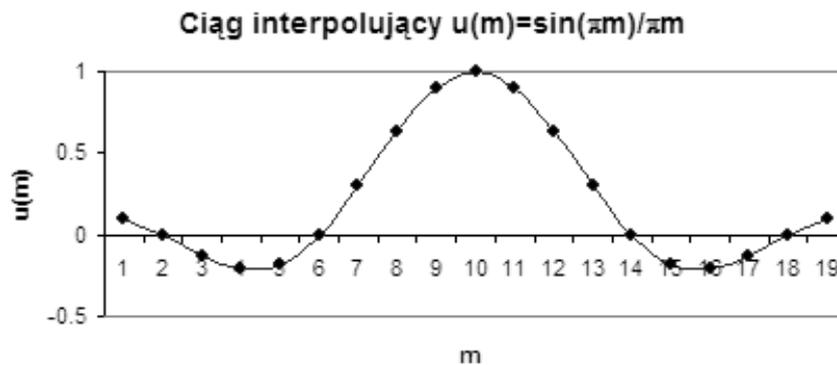


Fig. 6 Interpolation sinc function of  $m=19$

### **Algorithms of digital filtration and denosing (noise elimination)**

Due to fact, that each real signal analogue or discrete signal always apart from the signal which is under test – useful – wanted signal, the real signal contain unwanted component noise, so it is straightforward the noise reduction problem. After noise reduction the quotient of signal to noise is better so the dynamic of signal is better. It means that difference between useful (wanted, signal under measurement) and noise is bigger, so the signals of better quality, more dynamic. Increase of dynamic of signal can be achieved by:

- arithmetic synchronic (coherent) averaging of discrete signal in-time domain,
- arithmetic non-synchronic (non-coherent) averaging of discrete signal in-frequency domain (operation upon spectrum),
- weight averaging of signals in-time domain or in-frequency domain (for example MA, SOI, NOI filters),
- non-linear operations upon discrete signals in-time domain or in-frequency domain (for example median filter, adaptive filters, Poly-phase filters),

Exponential averaging is an example of low pass filter, which is equivalent to passive RC filter as it is presented in Fig.7.

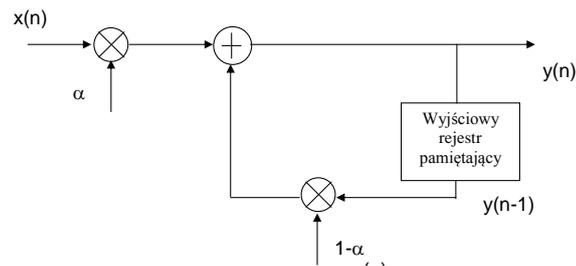


Fig 7 Block diagram of exponential averaging

Exponential averaging is a process of averaging in of discrete signal in-time domain and is proceed always on two succeeding samples of signal,  $x(n)$ .

System of Exponential averaging is described by following equation:

$$y(n) = \alpha \cdot x(n) + (1 - \alpha) \cdot y(n - 1)$$

where:  $\alpha$  is a weighting parameter of averaging from the range of  $(0, 1)$ , which determine the proportion between current and previous sample will appear at the output of a such filter. In this case averaging is leading to improve dynamic of a proceed signal (SNR) according to formula:

$$SNR_{EXP} = 10 \log_{10} \left( \frac{\alpha}{2 - \alpha} \right) [\text{dB}]$$

Lowering of  $\alpha$ -coefficient increase filter damping of averaging process and is equivalent to increasing of time constant in RC filter. If  $\alpha=1$  non modifications of input signal.

**Arithmetic averaging:** – averaging of successive elements of a set of samples in time domain of signal under filtration  $x(n)$  is denoted as Moving Average filter. MA filter is a low pass filter and name come from the result –of type of mathematical averaging operation. This a digital filter. a spectrum of such filter represents a set of non zero streak spectrum (non zero amplitude of spectrum lines) in the range from 0Hz do certain value called band value denoted as  $f_G$ . The process of MA filtration is a discrete convolution operation on signal tio be filtered  $x(n)$  and filtration set given by  $h(k)$  as follows:

$$y(n) = \sum_{k=0}^M h(k)x(n - k)$$

In general MA filter of M-order means that in each operation M samples are under averaging but coefficients of filtration set  $h(k)$  are even and equal  $1/M$ .

For example of 5 – order filter is multiplying 5 consecutive samples of the signal under filtration by filtration coefficients equal to:

$$h(0) = h(k) = \frac{1}{M} = 0.2 \Big|_{M=5}$$

As the result of MA filtering a band of signal under consideration would be lower. If this limitation is acceptable dynamic of signal becomes bigger and SNR coefficient is larger. (SNR Signal to Noise Ratio).

Median Filtration: is an example of nonlinear operation upon  $x(n)$  signal. An algorithm is similar to MA but instead averaging operation upon samples a sorting operation from the smallest to largest samples is done and next the median value of sorted  $y(n)$  is replacing set of samples used for sorting. It means that samples, which are actually sorted by means of median filter are such sorted that always the lowest and the largest values are on extreme positions and do not appear in filtrated  $Y(n)$  signal (set of new samples).

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## EXPERIMENT:

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### Task 1:

#### Spectral analysis of sampled analytic signal

The samples representing signal in time domain generate according to formula:

$$x(n) = A \cdot \sin(2\pi f n \Delta t)$$

Calculate values with a step  $\Delta t$  so you achieve 16 samples for one period ( $f_s = 16 \cdot f$ ), generate set of  $x(n)$  for only one period.  $A$ ,  $f$  parameter will be, suggested by experiment instructor.

Next:

- draw samples  $x(n)$  in-time domain,
- calculate *the spectrum complex signal using Fourier analysis of computational module available in the Tools / Data Analysis*
- *calculate and sketched  $Re\{X(k)\}$  and  $Im\{X(k)\}$*
- *calculate and sketched  $|X(k)|$  (module of a complex number) or amplitude spectrum of  $x(n)$*
- *calculate and sketched  $|\varphi(k)|$  (the argument of a complex number in degrees), ie, the signal phase spectrum  $x(n)$*

For engineering calculations used spreadsheet functions:

IMAGINARY() – the imaginary part of a complex number,

IMREAL() – the real part of a complex number ,

IMABS() – the module part the imaginary part of a complex number,  
 IMARGUMENT() argument  $\varphi$  the imaginary part of a complex number,  
 COMPLEX(.,.) – conversion of part real and imaginary to complex number,  
 STOPNIE(), RADIANY() – conversion og phase angle from radians to deg and vice versa

## TASK 2

### Spectral analysis of the recorded waveform by means of measurement card

In order to obtain passes in a string of discrete signal generator must be connected to the analogue input AI0 measuring card set available on the panel connector BNC\_1 laboratory. You can use the DataDSP data acquisition with A / D converter card to carry out sampling, synchronous measurement of the following analog signals from the digital generator:

- sinusoidal  $x_1(n)$  parameters will be recommended by instructor (amplitude and frequency)
- cosinusoidal  $x_2(n)$  parameters will be recommended by instructor (amplitude and frequency)
- rectangular  $x_3(n)$  o parameters will be recommended by instructor (amplitude and frequency and duty cycle),
- sinc  $x_4(n) = \text{sinc}(n)$  parameters will be recommended by instructor (amplitude and frequency)i
- rectangular impulse  $x_5(n)$  o parameters will be recommended by instructor (amplitude and frequency)
- white noise  $x_6(n)$  amplitude will be suggested by instructor.

In any case, keep the same sampling rate with a predetermined lead. Sets of data from the analogue-digital processing to import into a spreadsheet and make charts. Perform long sets of data fit the requirements of the Fourier analysis is available in the spreadsheet. Due to the fact that the spreadsheet is implemented FFT algorithm, and therefore required a numerical sequences of discrete signal in time domain must satisfy the condition that their length is equal to a multiple power series of 2 (... 8, 16, 32, ... .., 4096)

Next complete spectral analysis for collected sets of data  $x_1(n) \dots x_6(n)$  the same way as in Task 1. Sketch spectral amplitudes,  $|X(k)|$  and phase only  $|\varphi(k)|$ . Summaries the result in contexts od DFT analysis.

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**TASK 3*****Decimation process and interpolation of digital signals***

Complete the decimation operation of digital signal  $x(n)$  derived from the A / C at three different levels of decimation. (1, 3, 5).

In each case, draw diagram after decimation: amplitude of spectrum of these waveforms. Parameter and sampling frequency will be suggested by instructor.

In the summary task to interpret the shape of the spectrum over the decimation process and propose a solution to the problem of the observed after decimation.

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**ZADANIE 4*****Filtration and noise reduction***

Perform filtration and noise reduction of signal with a high content of random component (interference and noise) and non-periodic components. Waveforms sampled for measurement card-type earthquake, cardiovascular and rectangular from the generator to carry out the digital operation of discrete convolution (filtering) using the following filters:

- RC type filter for  $\alpha=0.3$ ,  $\alpha=0.6$ ,  $\alpha=0.9$
- MA type filter (moving average) 5, 9, 19 order.
- nonlinear filter of the third order.

After sketching, please make comments comparing: the spectrum before and after filtration process

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**FINAL REMARKS**


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Average value of discrete signal in the range

$$\bar{x} = \frac{1}{n_2 - n_1 + 1} \sum_{n=n_1}^{n_2} x(n)$$

The mean value of whole discrete signal

$$\bar{x} = \lim_{N \rightarrow \infty} \frac{1}{2N + 1} \sum_{n=-N}^N x(n)$$

The mean value of discrete periodic signals

$$\bar{x}_N = \frac{1}{N} \sum_{n=n_0}^{n_0+(N-1)} x(n), N - \text{period}$$

Energy of the whole discrete Signal

$$E_x = \sum_{n=-\infty}^{+\infty} x^2(n)$$

Average power in the certain range of discrete Signal

$$P_x = \overline{x^2} = \frac{1}{n_2 - n_1 + 1} \sum_{n=n_1}^{n_2} x^2(n)$$

Average power the entire signal (mean square value)

$$P_x = \bar{x} = \lim_{N \rightarrow \infty} \frac{1}{2N + 1} \sum_{n=-N}^N x^2(n)$$

Average power of discrete periodic signals

$$P_x = \overline{x^2}_N = \frac{1}{N} \sum_{n=n_0}^{n_0+(N-1)} x^2(n), N - \text{period}$$

RMS value of discrete signal

$$x_{RMS} = \sqrt{P_x}$$

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## LITERATURE AND OTHER RECOMMENDED MATERIAL

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1. Joseph McGhee, Wlodek Kulesza, M. Jerzy Korczyński, I. A. Henederson, Measurement Data Handling Theoretical Technique, Published by Technical University of Lodz, printed by: ACGM LODART S. A. Łódź, 2001, ISBN 83-7283-007-X, pages 267 vol. 1
2. Joseph McGhee, Wlodek Kulesza, M. Jerzy Korczyński, I. A. Henederson, Measurement Data Handling Hardware Technique, Published by Technical University of Lodz, printed by: ACGM LODART S. A. Łódź, 2001, ISBN 83-7283-007-8, pages 267 vol. 2
3. R.G.Lyons Wprowadzenie do cyfrowego przetwarzania sygnałów, WKŁ, Warszawa 1999
4. T.P. Zieliński Od teorii do cyfrowego przetwarzania sygnałów, Wydawnictwo ANTYKWA, Kraków 2002
5. T.P. Zieliński Zarys cyfrowego przetwarzania sygnałów. Od teorii do zastosowań Wydawnictwo WKŁ, Warszawa 2006
6. R.Plassche Scalone przetworniki analogowo-cyfrowe i cyfrowo analogowe, Wydawnictwo WKŁ, Warszawa 1997
7. C.M.Gilliam Ewers Zarys cyfrowego przetwarzania sygnałów, Wydawnictwo WKŁ, Warszawa 1999
8. D.Stranneby Cyfrowe przetwarzanie sygnałów metody, algorytmy, zastosowania, Wydawnictwo BTC, Warszawa 2004

### **Materiały dodatkowe:**

1. [www.dspguide.com](http://www.dspguide.com)
2. [www.analog.com/processors/learning/training/dsp\\_book\\_index.html](http://www.analog.com/processors/learning/training/dsp_book_index.html)

**Technical University of Lodz**  
**Department of Semiconductor and Optoelectronics**  
**Devices**

[WWW.DSOD.PL](http://WWW.DSOD.PL)

**ELECTRONIC MEASUREMENT LAB.**

<b>EXPERIMENT No:</b>	
<b>TITLE:</b>	

<b>Laboratory Group</b>		<b>Telecommunication and Computer Science</b>	
<b>no.</b>	<b>Name and Surname</b>	<b>Student ID</b>	
<b>1</b>			
<b>2</b>			
<b>3</b>			
<b>4</b>			

<b>Lecturer:</b>	
<b>Date of experiment:</b>	
<b>Date of report presentation:</b>	
<b>Mark:</b>	
<b>Remarks:</b>	